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Coherent Dynamics and Instability of Power Grids

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Abstract—We describe a structure of model for short-term swing dynamics of multi-machine power grids, exhibiting an instability phenomenon termed the Coherent Swing Stability (CSI). This model is based on the swing equations with a linear term representing the interconnection between synchronous machines in a power grid. We analyze the New England 39-bus test system exhibiting CSI and show that the phenomenon happens in the dynamical system with one nonlinear mode that is weak relative to many linear oscillatory modes.

1. Introduction

Coupled swing dynamics in a population of synchronous machines are of vital importance for power grid stability. The so-called transient stability analysis is associated with the ability of power grid to maintain synchronism when subjected to a large disturbance [1]. Loss of transient stability is recognized as one cause of large blackouts such as the September 2003 blackout in Italy [2]. Transient stability is mainly governed by electromechanical oscillations of synchronous machines in short-term regime (0 to 10 seconds) and is mathematically investigated by the so-called swing equations. Analysis of the swing equations is hence needed for prevention of transient instability (see e.g. [3, 4, 5]).

In [6, 7] we uncovered a new phenomenon in short-term swing dynamics of multi-machine power grids, which we termed the *Coherent Swing Instability* (CSI), based on the notion of instability occurring for general oscillatory systems described in [8, 9, 10, 11, 12]. CSI is an undesirable and emergent phenomenon of synchronous machines in a power grid, in which machines in a subset of the grid *coherently* lose synchronism with the rest of the grid after being subjected to a finite and local disturbance. This phenomenon gives the dynamical mechanism that explains how local plant mode oscillation, inter-area mode instability, and multi-swing instability interact to destabilize a power grid.

The purpose of this paper is to describe a structure of mathematical model for short-term swing dynamics, exhibiting CSI, in general power grids. This model is the swing equations with a linear term representing the interconnection between synchronous machines in a power grid. We analyze the New England (NE) 39-bus test system [13, 14] for which the CSI phenomenon is reported in [6, 7]. The analysis is based on normal mode decomposition that is a well-known technique in vibration. Thus we show that the phenomenon occurs in the dynamical system with one nonlinear mode that is weak relative to linear oscillatory modes. This structure is equivalent to that in the simple loop grid studied in [6].

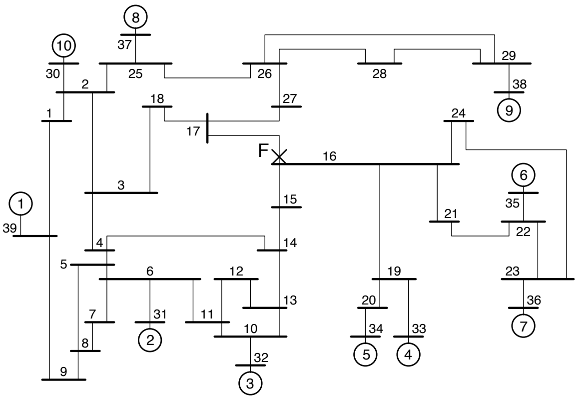

Figure 1: The New England 39-bus test system [13, 14]

2. Numerical Simulation

In this section we review the CSI phenomenon in the New England (NE) 39-bus test system shown in Fig. 1. The NE grid consists of 10 synchronous generators, 39 buses, loads, and ac transmission lines.

2.1. The Swing Equations

We introduce the equations of motion for the NE grid. To do so we assume that bus 39 is the infinite bus[1] in Fig. 1. The short-term swing dynamics of generators 2–10 are rep-

[1]An infinite bus is a voltage source of constant voltage and constant frequency.

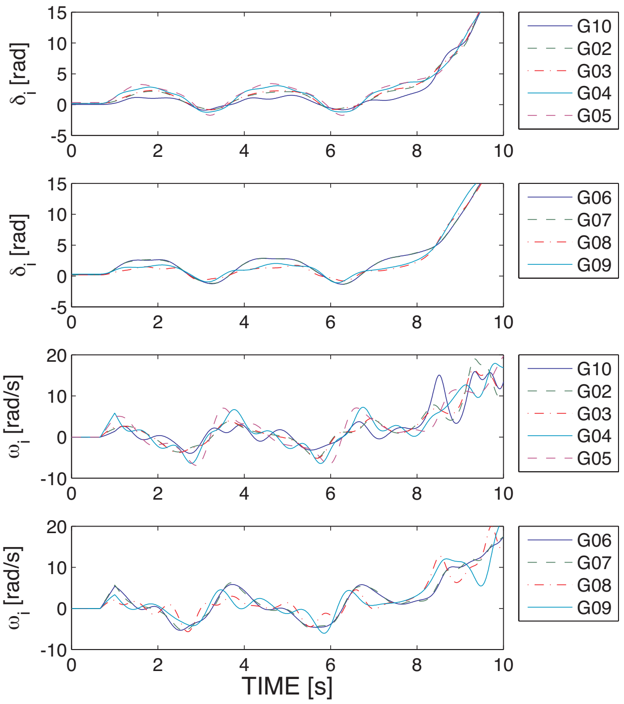

Figure 2: Coupled swing dynamics in the New England 39-bus test system

resented by the swing equations [1, 14]:

$$\left.\begin{aligned} \frac{H_i}{\pi f_{\mathrm{s}}}\frac{\mathrm{d}^2\delta_i}{\mathrm{d}t^2} + D_i\frac{\mathrm{d}\delta_i}{\mathrm{d}t} &= P_{\mathrm{m}i} - G_{ii}E_i^2 \\ - \sum_{j=1, j\neq i}^{10} E_iE_j\{G_{ij}\cos(\delta_i-\delta_j) &+ B_{ij}\sin(\delta_i-\delta_j)\}, \end{aligned}\right\} \quad (1)$$

where the integer label $i = 2, \ldots, 10$ denotes generator i. The variable δ_i is the angular position of rotor in generator i with respect to bus 1 and is in radian. We set the variable δ_1 to a constant, because bus 1 is assumed to be the infinite bus. The parameters f_{s}, H_i, D_i, $P_{\mathrm{m}i}$, E_i, G_{ii}, G_{ij}, and B_{ij} are in per unit system except for H_i and D_i in second, and for f_{s} in Helz. The mechanical input power $P_{\mathrm{m}i}$ to generator i and the internal voltage E_i of generator i are normally constant for transient stability analysis [1]. The parameter H_i is the per unit time inertia constant of generator i, and D_i its damping coefficient. The parameter G_{ii} is the internal conductance, and $G_{ij} + \mathrm{j}B_{ij}$ is the transfer impedance between generators i and j. They are the parameters that change with network topology changes. Electrical loads are modeled as passive impedances.

2.2. Coherent Swing Instability [6, 7]

We numerically simulate coupled swing dynamics of generators 2–10 in the NE grid. The voltage E_i and the initial condition $(\delta_i(0), \omega_i(0) = 0)$ for generator i are calculated using power flow computation. The inertia constant H_i is the same as in [14]. For the simulation we use the following load condition: $P_{\mathrm{m}i}$ and constant power loads are 50% at their rating. The damping D_i is fixed at 0.005 s for each generator. The elements G_{ii}, G_{ij}, and B_{ij} are calculated using the data in [14] and the result of power flow computation. We use the following fault condition: each generator operates at a steady condition at $t = 0$ s, a three-phase fault happens at point F near bus 16 at $t = 1\,\mathrm{s} - 20/(60\,\mathrm{Hz}) = 2/3$ s, and line 16–17 trips at $t = 1$ s. The fault duration is 20 cycles of a 60-Hz sine wave. The fault is simulated by adding a small impedance (10^{-7}j) between bus 16 and the ground.

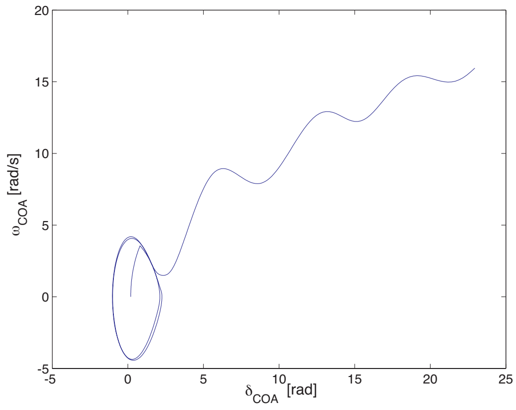

Figure 3: Collective motion of coupled swing dynamics in the New England 39-bus test system

Figure 2 shows the time responses of angular position δ_i and its derivative $\omega_i = \mathrm{d}\delta_i/\mathrm{d}t$ of generator i. Before $t = 2/3$ s (the onset time of the fault), each generator operates at a steady condition. In the fault duration from $t = 2/3$ s to 1 s, all the generators 2–10 accelerate from their steady conditions. After the line trip at $t = 1$ s, they respond in an oscillatory manner. These oscillations are bounded during the period from $t = 1$ s to 8 s and then begin to grow coherently, that is, every generator loses synchronism with the infinite bus at the same time. This corresponds to the growth of amplitude of inter-area mode oscillation between the NE grid and the infinite bus, namely, the outside of the grid. This is typical of the CSI phenomenon.

In [6] we showed that CSI involves the divergent motion in the projection of full-system dynamics onto the state plane of collective variables. The collective variables correspond to the well-known COA (Center-Of-Angle) variables in [13, 14]. For the NE grid, the COA δ_{COA} and its time derivative ω_{COA} are defined as

$$\delta_{\mathrm{COA}} = \sum_{i=2}^{10}\frac{M_i}{M}\delta_i, \quad \omega_{\mathrm{COA}} = \frac{\mathrm{d}\delta_{\mathrm{COA}}}{\mathrm{d}t} = \sum_{i=2}^{10}\frac{M_i}{M}\omega_i, \qquad (2)$$

where $M_i = H_i/(\pi f_{\mathrm{s}})$ and $M = \sum M_i$. The variables δ_{COA} and ω_{COA} describe the averaged motion of all the generators in the grid. Fig. 3 plots the trajectory of (1) showing the phenomenon in Fig. 2, on δ_{COA}–ω_{COA} plane. The trajectory starts near the origin at time 0 s, makes a couple of almost periodic loops around the initial point, and finally diverges.

3. Analytical Studies

In this section we describe a structure of the swing equations with the help of partial linearization, linear normal mode, and internal resonance, thereby clarifying dynamics behind the phenomenon.

3.1. Model System and Partial Linearization

Consider a power grid consisting of N synchronous generators, loads, the infinite bus, and an ac transmission network with arbitrary topology. We assume that electric loads in the grid are represented by constant active/reactive power and are modeled by lumped impedances. We start our argument by introducing the swing equations:

$$M_i\frac{\mathrm{d}^2\delta_i}{\mathrm{d}t^2}+D_i\frac{\mathrm{d}\delta_i}{\mathrm{d}t} = P_{\mathrm{m}i}-P_{\mathrm{e}i}(\delta_0,\dots,\delta_N), \tag{3}$$

with

$$\begin{aligned} P_{\mathrm{e}i} &= G_{ii}E_i^2 \\ &+\sum_{j=0,j\neq i}^{N} E_iE_j\{G_{ij}\cos(\delta_i-\delta_j)+B_{ij}\sin(\delta_i-\delta_j)\}, \end{aligned} \tag{4}$$

where $i=0,\dots,N-1$ is the integer index labeling generator i. The variable δ_i represents the angular position of generator i with respect to the infinite bus. δ_N is the angular position of the infinite bus is assumed to be zero without the loss of generality. The parameters $M_i(>0)$, $D_i(>0)$, $P_{\mathrm{m}i}$, E_i, G_{ij}, and B_{ij} are constant.

We now simplify (3) in order to analyze the phenomenon of interest, where the angle differences between individual generators stay small for all time. Assume that the difference between any two swings $\delta_i(t)$ and $\delta_j(t)$ $(i,j=0,\dots,N-1)$ is small: for a small parameter ε,

$$\delta_i(t)-\delta_j(t) = \varepsilon z_{ij}(t), \tag{5}$$

where $z_{ij}(t)$ is the time-dependent function containing high harmonic components. Substituting the small difference $\varepsilon z_{ij}(t)$ into the interconnection term of (3), we have the following first-order approximation:

$$\begin{aligned} M_i\frac{\mathrm{d}^2\delta_i}{\mathrm{d}t^2}+D_i\frac{\mathrm{d}\delta_i}{\mathrm{d}t}+\frac{\partial}{\partial\delta_i}\mathcal{U}_i(\delta_i) &= -\sum_{j=0}^{N-1}A_{ij}\delta_j \\ &= -A_{ij}\delta_j, \end{aligned} \tag{6}$$

where we use the Einstein notation. The potential function $\mathcal{U}_i(\delta_i)$ represents the interaction of generator i with the infinite bus and is given by

$$\begin{aligned} \mathcal{U}_i(\delta_i) &= -(P_{\mathrm{m}i}-E_iE_jG_{ij})\delta_i \\ &\quad +E_iE_N(G_{i,N}\sin\delta_i-B_{i,N}\cos\delta_i). \end{aligned} \tag{7}$$

The matrix $\mathsf{A}=\{A_{ij}\}$ represents the interaction between generators in the grid. The matrix A is real and symmetric, defined as

$$A_{ij}=\begin{cases} E_iE_jB_{ij} & \text{if } i\neq j, \\ -\displaystyle\sum_{k=0,k\neq i}^{N-1}E_iE_kB_{ik} & \text{if } i=j. \end{cases} \tag{8}$$

For the matrix A with eigenvalues $\lambda_0\leq\lambda_1\leq\cdots\leq\lambda_{N-1}$, it is stated in [15] that A is positive semidefinite (λ_i for any i is non-negative), and λ_0 is identically zero with the unit eigenvector $\boldsymbol{u}_0=(1,1,\dots,1)^{\mathrm{T}}/\sqrt{N}$ (where T denotes the transpose operation of vector and matrix). It is easily proved that any two eigenvectors $\boldsymbol{u}_i$ and $\boldsymbol{u}_j$ for different eigenvalues λ_i and λ_j are orthogonal. These properties of A hold for arbitrary power grid, unless it does not have any active elements such as phase shifter and power electronics device.

3.2. Model Decomposition

Now we decompose (6) using linear normal modes determined by the inertial and interconnection terms. Consider the linear system obtained by picking up the inertial and interconnection terms from (6):

$$M_i\frac{\mathrm{d}^2\delta_i}{\mathrm{d}t^2} = -A_{ij}\delta_j, \tag{9}$$

By defining the new variable $x_i=\sqrt{M_i}\delta_i$, we re-write the above linear system as

$$\frac{\mathrm{d}^2x_i}{\mathrm{d}t^2} = -\frac{A_{ij}}{\sqrt{M_iM_j}}x_j, \tag{10}$$

Since the inverse of matrix $\sqrt{\mathsf{M}}=\mathrm{diag}(\sqrt{M_0},\dots,\sqrt{M_{N-1}})$ is positive definite, the matrix $\mathsf{K}=\sqrt{\mathsf{M}}^{-1}\mathsf{A}\sqrt{\mathsf{M}}^{-1}=\{A_{ij}/\sqrt{M_iM_j}\}$ is still real, symmetric, and positive semidefinite and has the (real) non-negative eigenvalues $\Omega_0^2\leq\Omega_1^2\leq\cdots\leq\Omega_{N-1}^2$ with unit eigenvectors $\boldsymbol{v}_0,\dots,\boldsymbol{v}_{N-1}$. For the eigenvalue $\Omega_0=0$, the unit eigenvector $\boldsymbol{v}_0$ is $(\sqrt{M_0},\dots,\sqrt{M_{N-1}})^{\mathrm{T}}/\sqrt{M}$, where $M=M_i$. Using the matrix $\mathsf{V}=\{V_{ij}\}=[\boldsymbol{v}_0\boldsymbol{v}_1\dots\boldsymbol{v}_{N-1}]$, we can diagonalize the linear system as follows:

$$\frac{\mathrm{d}^2q_i}{\mathrm{d}t^2} = -\Omega_i^2q_i, \tag{11}$$

In the above derivation, we use the change of variables $q_i=(\mathsf{V}^{\mathrm{T}})_{ij}x_j=V_{ij}^{\mathrm{T}}x_j=V_{ji}x_j$ and $x_i=V_{ij}q_j$. By applying the same change of variables to (6), we have the following equations:

$$\left.\begin{aligned} \frac{\mathrm{d}q_i}{\mathrm{d}t} &= p_i, \\ \frac{\mathrm{d}p_i}{\mathrm{d}t} &= -\tilde{D}_{ij}p_j-T_{ij}^{\mathrm{T}}\frac{\partial}{\partial\delta_j}\mathcal{U}_j(T_{jk}q_k)-\Omega_i^2q_i, \end{aligned}\right\} \tag{12}$$

where $\tilde{D}_{ij} = V_{ik}^{\mathrm{T}} D_k / M_k V_{kj}$ and $T_{jk} \triangleq V_{jk} / \sqrt{M_j}$. Equation (12) is regarded as the dynamical system of N linear modes coupled via the terms of nonlinearity and dissipation. This system is applicable to any power grid. Note that the zeroth mode variables (q_0, p_0) associated with zero eigenvalue $\Omega_0^2 = 0$ are proportional to the COA variables $(\delta_{\mathrm{COA}}, \omega_{\mathrm{COA}})$:

$$(q_0, p_0) = \sqrt{M}(\delta_{\mathrm{COA}}, \omega_{\mathrm{COA}}). \tag{13}$$

3.3. On the Mechanism

We describe the dynamical structure of the NE grid exhibiting the CSI shown in Fig. 2. In the NE grid, the magnitude of interaction of individual generators with the infinite bus is much smaller than that of interaction between generators. This is numerically confirmed by estimating the L^1-norm of the nonlinear function $\partial \mathcal{U}_i / \partial \delta_i$ and the induced one-norm of the matrix A: see [7] for details. Since the damping coefficient D_i is relatively small, by introducing a small parameter ϵ, we can rewrite (12) as follows:

$$\left.\begin{aligned} \frac{\mathrm{d}q_i}{\mathrm{d}t} &= p_i, \\ \frac{\mathrm{d}p_i}{\mathrm{d}t} &= \epsilon f_i(q_0, \ldots, q_{N-1}, p_0, \ldots, p_{N-1}) - \Omega_i^2 q_i, \end{aligned}\right\} \tag{14}$$

where

$$\epsilon f_i = -\tilde{D}_{ij} p_j - T_{ij}^{\mathrm{T}} \frac{\partial}{\partial \delta_j} \mathcal{U}_j(T_{jk} q_k). \tag{15}$$

From the above argument, the linear term $-\Omega_i^2 q_i$ determined by A is dominant in (14). However, for the zeroth mode with zero eigenvalue, the nonlinear term ϵf_0 becomes dominant. We call this zeroth mode the *nonlinear* one. Thus we can say that the dynamical system (6) or (12) has one nonlinear mode that is weak relative to linear oscillatory modes.

Finally we discuss the CSI with dynamical systems theory close to resonance [16]. Consider the equations for the nonlinear mode in (14):

$$\left.\begin{aligned} \frac{\mathrm{d}q_0}{\mathrm{d}t} &= p_0, \\ \frac{\mathrm{d}p_0}{\mathrm{d}t} &= \epsilon f_0(q_0, \ldots, q_{N-1}, p_0, \ldots, p_{N-1}). \end{aligned}\right\} \tag{16}$$

The variable p_0 is *slow* due to the presence of small parameter ϵ, and q_0 is *semi-fast* because it moves fast except for the vicinity of q_0-axis, namely, $p_0 = 0$. The set of q_0-axis is called the *resonant surface* and is a $(2N-1)$-dimensional subspace in the full phase space. When recalling (12), we see in Fig. 3 that the trajectory projected onto p_0–q_0 plane starts on the resonant surface and finally escapes it. Thus we say that CSI occurs when dynamics of (6) escape from the vicinity of the resonant surface.

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References

[1] P. Kundur, *Power System Stability and Control*. McGraw-Hill, 1994.

[2] S. Corsi and C. Sabelli, "General blackout in Italy Sunday September 28, 2003, h. 03:28:00," in *Proceedings of the IEEE PES General Meeting*, vol. 2, Denver, USA, June 2004, pp. 1691–1702.

[3] N. Kopell and R. B. Washburn Jr., "Chaotic motions in the two-degree-of-freedom swing equations," *IEEE Transactions on Circuits and Systems*, vol. CAS-29, no. 11, pp. 738–746, November 1982.

[4] F. M. A. Salam, J. E. Marsden, and P. P. Varaiya, "Arnold diffusion in the swing equations of a power system," *IEEE Transactions on Circuits and Systems*, vol. CAS-31, no. 8, pp. 673–688, August 1984.

[5] Y. Ueda, T. Enomoto, and H. B. Stewart, "Chaotic transients and fractal structures governing coupled swing dynamics," in *Applied Chaos*, J. H. Kim and J. Stringer, Eds. London: John Wiley & Sons, 1992, ch. 8.

[6] Y. Susuki, I. Mezić, and T. Hikihara, "Global swing instability of multimachine power systems," in *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, December 9–11 2008, pp. 2487–2492.

[7] ——, "Coherent swing instability of power grids," (in preparation).

[8] I. Mezić, "Dynamics and control of large-scale molecular motion," in *Preprint of IFAC Symposium*.

[9] ——, "On the dynamics of molecular conformation," *Proceedings of the National Academy of Sciences of the USA*, vol. 103, no. 20, pp. 7542–7547, 2006.

[10] B. Eisenhower and I. Mezić, "A mechanism for energy transfer leading to conformation change in networked nonlinear systems," in *Proceedings of the 46th IEEE Conference on Decision and Control*, New Orleans, USA, December 2007, pp. 3976–3981.

[11] ——, "Actuation requirements of high dimensional oscillator systems," in *Proceedings of the 2008 American Control Conference*, Seattle, USA, June 2008, pp. 177–182.

[12] P. Du Toit, I. Mezić, and J. Marsden, "Coupled oscillator models with no scale separation," *Physica D*, vol. 238, no. 5, pp. 490–501, March 2009.

[13] T. Athay, R. Podmore, and S. Virmani, "A practical method for the direct analysis of transient stability," *IEEE Transactions on Power Appratatus and Systems*, vol. PAS-98, no. 2, pp. 573–584, March/April 1979.

[14] M. A. Pai, *Energy Function Analysis for Power System Stability*. Kluwer Academic Pub., 1989.

[15] B. Avramović, P. V. Kokotović, J. R. Winkelman, and J. H. Chow, "Area decomposition for electromechanical models of power systems," *Automatica*, vol. 16, pp. 637–648, 1980.

[16] V. I. Arnold, V. V. Kozlov, and A. I. Neishtadt, *Mathematical Aspects of Classical and Celestial Mechanics*, 3rd ed., ser. Encyclopaedia of Mathematical Sciences. Heidelberg Germany: Springer-Verlag, 2006, vol. 3, chapter 6.